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Mathematical Representations and Pedagogical Content Knowledge: An Investigation of Prospective Teachers' Development

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A study was carried out involving K-8 teacher candidates enrolled in an elementary mathematics methods course to investigate and document their thinking as they plan for mathematics instruction. The teacher candidates submitted lesson plans at three intervals during a semester-long methods course, which were coded based on the planned use(s) of mathematical representations. Analysis of the data revealed trends in the choices of representations. Recommendations are presented highlighting the potential benefits of incorporating the knowledge base on mathematical representations into a mathematics methods course and a discussion ensues on the development of these teacher candidates' pedagogical content knowledge through their choice(s) of mathematical representations in their lesson planning.

Introduction

A primary goal of a methods course is to prepare future teachers to teach the subject matter in effective and engaging ways such that student understanding is maximized. In other words, the authors posit that methods courses provide opportunities for teacher candidates to develop and further their pedagogical content knowledge. In an attempt to document and explore how K-8 teacher candidates represent mathematical ideas in ways that are understandable to students, a study was carried out in which K-8 teacher candidates submitted lesson plans at three intervals during a semester. Serving as a lens during the coding and analyses of these lesson plans were the five representations defined by Lesh, Post, & Behr (1987). Of interest to the researchers was what representation(s) these teacher candidates might choose to use as they plan for teaching a mathematics topic in the most effective way as possible; that is, so that student understanding is maximized. Also of interest was the potential emergence of trends in these teacher candidates' choices of representations in their lesson planning as they progressed through their semester-long elementary mathematics methods course. After analyzing the lesson plans of these teacher candidates at three intervals, the researchers were able to document the planned choices of and trends in both teacher and student use of various representations within their lesson plans. These choices and trends will be described and a discussion will follow presenting the potential benefits of incorporating the knowledge base on mathematical representations into a mathematics methods course to contribute to the development of K-8 teacher candidates' pedagogical content knowledge.

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Research on pedagogical content knowledge

Many researchers have documented the various type(s) of knowledge that are needed by teachers. In particular, Shulman and Quinlan (1996) define pedagogical content knowledge as the transformation of a teacher's "own content knowledge into pedagogical representations that connect with the prior knowledge and disposition of the learner" (p. 409). Shulman (1986) defines pedagogical content knowledge as the ability to represent ideas in ways that are understandable to students. Shulman argues that teachers need to know things like what topics children find interesting or difficult and what the representations are that are most useful for teaching a specific content idea. Shulman (1987) further elaborates on pedagogical content knowledge as the capacity "to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students" (p. 15).

For Carter (1990), pedagogical content knowledge represents an attempt to determine what teachers know about their subject matter and how they translate that knowledge into classroom curricular events. Doyle (1992) contends that this capacity distinguishes a teacher from a non-teaching specialist; for example, "Knowing biology is necessary, but certainly not sufficient, to know how to represent biological content to students in a teaching situation" (p. 498).

Kennedy (1998) offers that recitational knowledge; that is, the traditional mode of defining facts and terminology is not sufficient for teaching. Instead, other types of knowledge are needed such as the conceptual understanding of subject matter as well as pedagogical content knowledge. "Teachers need to be able to respond to questions and hypotheses that they might not have anticipated, provide students with guidance when they get in over their heads, clarify confusions, and ensure that misconceptions are not perpetuated" (Kennedy, 1998, p. 252). Additionally, Ball and Bass (2000) posit that in order to make mathematical knowledge usable, teachers must know content sufficiently and flexibly such that it can be used within a wide variety of contexts.

Research on representations

Lesh, Post, & Behr (1987) offer five distinct different representations of mathematical ideas, namely, concrete (manipulatives), language, symbolism (notation), semi-concrete (pictorial), and contextual (real-world situations). As an example to the reader, the researchers present their interpretation of these five representations by considering how a teacher might represent the concept of 65 (see figure 1). A teacher could use counters or money to represent 65 concretely. A teacher could use language such as, "Sixty-five is five less than seventy" to represent this concept or use the notation, "100-35= 65" to represent this concept symbolically. Similarly, a teacher might use a set model to represent 65 pictorially or a teacher might offer the real-life context of the cost of an item or the age at which one retires to assist a student in understanding the concept of 65. According to Lesh, Post, & Behr (1987), strengthening the ability to move between and among representations improves the growth of students' understanding of mathematical concepts.

In addition, NCTM (2000) notes, "when students gain access to mathematical representations and the ideas they represent, they have a set of tools that significantly expand their capacity to think mathematically" (p.67). Representations can then be thought of as *thinking tools* to communicate mathematical ideas. The model that Lesh, Post, and Behr (1987)

have developed of external representations allows for teachers and students to have a common language for communicating internal mathematical ideas.

Methodology

The study involved thirty-one elementary education majors enrolled in a field-based mathematics methods course at a large southwestern university. All of these individuals had successfully completed a mathematics content course, a prerequisite to the methods course. Two of the participants were male and 29 were female. Throughout the semester, the teacher candidates were taught in a constructivist manner (using manipulatives, technology, problem solving, hands-on exploration, writing, discourse, making real-world connections, etc.) consistent with the reform standards (NCTM, 1991, 2000). During the methods course, the teacher candidates simultaneously participated in the course's field component, where they were observed an elementary classroom daily.

At the beginning of the semester, the researchers compiled a list of eight mathematical topics that would be explored throughout the methods course and which represented typical K-8 mathematics topics these teacher candidates would be expected to teach. These topics included: multiplication of fractions, division of fractions, area of a circle, area of a trapezoid, area of a parallelogram, perimeter of polygons, addition and subtraction of integers, and mean. The topics, along with a corresponding and appropriate grade level, were written individually on index cards prior to class and one index card was distributed to each teacher candidate during the beginning portion of the semester. The teacher candidates were then instructed to develop and submit a lesson plan one week later reflecting what they considered to be an effective way to teach that particular topic; that is, such that student understanding would be maximized. The teacher candidates were encouraged, but not required, to seek out resources such as books, the Internet, speak to inservice teachers, etc. to assist them in this assignment. The teacher candidates were asked that when designing their lesson plans, they include along with the topic and grade level, any materials, the procedure, a closure, and the source (if any) of their ideas. For coding purposes, the lesson plans submitted at this initial juncture were labeled as "initial" lesson plans.

One week later, after completing and submitting their individual lesson plans, the teacher candidates were placed in groups with those sharing the same topic and, with their classmates, were asked to reach a group consensus on how to best teach their shared topic. During this time, the teacher candidates were encouraged to share their individual ideas described in their lesson plans and to justify to their classmates why they believed their method and approach to teaching this topic was effective. The teacher candidates were given one hour of class time to engage in discussion and to reach a unanimous decision. Additionally, the groups were asked to identify a note-taker, who would capture the finalized ideas on paper and a typist, who would electronically submit the group's finalized lesson plan one week later using the aforementioned format. For coding purposes, the lesson plans submitted at this middle juncture were labeled as "group" lesson plans.

While the teacher candidates were sharing their ideas and deliberating over how to best teach their given topic, each group was videotaped and voice recorded in an attempt to capture their conversational journey as they first each presented their individual arguments and justifications for how to best teach their topic and then concluding with how the group came to a consensus on how to teach the topic.

At the end of the semester, the teacher candidates were asked to individually submit one final lesson plan, again using the previously described format, but this time detailing how to best teach a K-8 mathematical topic of their choice. Topics chosen by the teacher candidates included a wide range of grade levels and concepts, such as addition, area of a circle, counting, fractions, graphs, least common multiple, money, multiplication facts, probability, rate of acceleration, rounding, shapes, solids, subtraction, tessellations, and time. For coding purposes, the lesson plans submitted at this final juncture were labeled as "final" lesson plans.

Upon receiving the teacher candidates' lesson plans at each of the three intervals, the researchers painstakingly coded the lessons plans noting each time one of the five aforementioned representations (Lesh, Post, Behr, 1987) was used. However, after coding several of the "initial" lesson plans and recognizing the apparent emerging gap between the number of times the teacher was using a representation versus the students, the researchers refined their coding process by delineating between each time a representation was used by the teacher in the lesson plan versus students for later possible comparison purposes.

Additionally, during the coding process, the researchers sub-divided the language representation into two sub-categories hereto refereed to as, L_1 and L_2. L_1 referred to language that is used to talk about mathematical procedures and defining mathematical terms whereas L_2 referred to mathematics discourse; that is, rich and thoughtful discussion about the mathematics and questions that encourage higher-level thinking as well as explaining, justifying, questioning, and challenging (Wood, 1999). The researchers felt this sub-categorization of language representation was desirable, as the researchers were curious as to the extent to which these teacher candidates adhered to the NCTM *Standards* (2000), which strongly advocate discourse in the mathematics classroom.

As an example to the reader, consider the following portion of a "group" lesson plan submitted by a group of teacher candidates on the topic of perimeter of polygons. The lesson, designed for third graders, begins:

> "Discuss the characteristics of a polygon with the class. Explain that a polygon has straight lines with no arcs.
>
> Pass out a geoboard to each student or pair of students.
>
> Have the students construct various polygons on the geoboards emphasizing the attributes of a polygon.
>
> As the students are working, tape large shapes onto the floor, one shape per group of students. Distribute colored construction paper and instruct students to trace their foot and cut it out.
>
> Divide the class into groups explaining that they will use their paper foot to determine the distance around the shape. They are to find out how many of their paper feet will go around the shape..."

The first two sentences above were coded as the teacher using language (L_1) as a representation of the idea of a polygon. The third sentence was not coded since it is a logistical aspect of the lesson plan. The fourth sentence was coded as students using a concrete

representation of a polygon along with them using L_1. The fifth and sixth sentences were coded as pictorial representations of polygons that the students would be potentially using to find perimeters. The seventh and eighth sentences were coded as the teachers' use of L_1 to represent the notion of perimeter.

Listed in Figure 2 are the percentages of times the various representations were used within the lesson plans submitted at each of the three intervals. Figure 3 presents these same percentages, but separated out by teacher versus student use.

Discussion of the findings

The researchers caution that the reader keep in mind that these lesson plans are hypothetical teaching acts, as these lesson plans were not enacted in an actual classroom setting. Thus, the teaching approaches and ideas presented in the lesson plans provide us with insight into these teacher candidates' thinking about teaching mathematics.

After analyzing the data, language was found to be the most used representation in the lesson plans submitted by the teacher candidates at all three intervals. Moreover, the use of L_1 (mathematical procedures and/or information/fact-giving) heavily outweighed the use of L_2 (mathematical discourse) and was one of the most used representations overall. Also, in comparing the initial lesson plans to the final lesson plans, the use of L_1 increased from 33% to 46%. A disappointing trend was that L_2 was the least used representation overall. One possible explanation for this might be the reluctance, apprehension, or perhaps lack of recognition of its importance, on the part of these teacher candidates, to plan for and encourage rich discussion and discourse in their lesson planning, despite this type of language being modeled and encouraged regularly in their mathematics methods course. Also noted was the infrequent use of and decreasing trend in the contextual (real-world) representation, again, despite its frequent modeling by the methods instructor and the discussion of its importance during class.

Although its use sharply declined (32% to 14%) when comparing the initially submitted lesson plans to those submitted at the end of the semester, symbolism, like language, was also a heavily used representation. Little change was noted in the semi-concrete (pictorial representation), decreasing ever so slightly over the course of the semester from 16% to 15%.

Although its use did increase from 10% to 18%, prior to coding the lesson plans, the researchers anticipated a higher occurrence of the concrete representation in all three of the submitted lesson plans, primarily due to the fact that manipulatives were used in every class meeting of both the methods course and the required prerequisite mathematics content course. This assumption was also supported by the fact that during the videotaping of the teacher candidates' group discussions, they repeatedly voiced the importance of using manipulatives to provide a "hands-on approach." For example, an individual in the group whose topic was mean suggested to her classmates that the students should all have "some sort of manipulative for them to actually see it [the mean] with their hands." In another group whose topic was the perimeter of polygons, one student reiterated her group members' sentiments stating, "We like hands-on." Consequently, the group then unanimously agreed to use geoboards to teach this topic as another individual in this same group confirmed, "These [geo] boards help out a lot." There is a disjuncture in the findings in comparing the written documents of planned instruction to the oral discussions among the groups when sharing ideas for planning instruction.

Upon comparing the percentages of planned uses of the various representations by the teacher as opposed to the students, (see figure 3), the researchers noted that although student use of language increased over time (from 7% to 19%) it was the teacher who used the language

representation far more often in the lesson plans. Additionally, teacher use of L_1 outweighed student use of L_1 and, although it was the least often used representation, L_2 was used more often by the students than by the teacher. The heavy use of L_1 by the teacher in the lesson plans could be attributed to the fact that, in the lesson plans, the teacher candidates specified how the teacher would have to "introduce the formula" and "explain' and "tell" students how a formula was derived or "how it worked." One student whose topic was the area of a trapezoid told his group, "The actual formula will come later in the lesson. You're not gonna blast them with the formula right away." When the teacher candidates were able to collaborate and negotiate with their classmates in their respective groups, the student use of L_2 increased as the teacher candidates began to allow for the students to explain, apply, justify, and "figure out for themselves" the formulas necessary to understand their topic.

In regard to the other representations, student use of the concrete representation outweighed teacher use primarily because, as noted during the coding of the lesson plans, the students were actively engaged in using the manipulatives, as opposed to simply watching the teacher demonstrate some concept using the manipulative solely. During the second and third intervals, student and teacher use of symbolism and the contextual representation were somewhat balanced while there existed a transition from teacher-use to a more balanced teacher/student-use for the semi-concrete (pictorial) representation.

Figure 3 also illustrates the overall percentage of time the representations were planned to be used by the student (S) as opposed to the teacher (T). In the initial lesson plans, the researchers noted that the teacher (66% vs. 34%) used the representations approximately two-thirds of the time. In the group lesson plans, the teacher and the students were using the representations almost equally (47% vs. 53%) whereas in the final lesson plans, the teacher and the students were using the representations equally (50% vs. 50%). The researchers posit that this more balanced approach to planning for their teaching, where both the teacher and the students are equally engaged in the mathematics, could be attributed to the modeling that these teacher candidates observed in their mathematics methods class, where they were equally responsible for and engaged in understanding the material. Another contributing factor could be the added observation of the use of manipulatives in the elementary classrooms that they observed and participated in during the field component of their methods course. At some point during this time, as these teacher candidates prepared to become teachers, they may have begun the process of valuing the use of concrete materials when exploring mathematics concepts.

Other findings

After reading the transcriptions of the videotaped group discussion, the researchers noted that not only did the teacher candidates negotiate, question, challenge, and justify their preferences for how and why they chose to teach the topic, but the discussions also included the logistics of teaching. For example, within some groups, the conversation at times focused on such logistical issues as the use of worksheets, grouping, and time limitations. Their comments included:

> Maybe we should just do a worksheet...we could give them a few problems.
>
> "How many examples do we need to use on one paper? Do you think 4? Is this sufficient?"

"We should put a time limit; for fifth graders, maybe 45 minutes. Do we want to do groups or individuals?"

"We need to decide what will happen on each day."

Some of these logistical issues demonstrated the teacher candidates' growth in other areas of pedagogical knowledge; in particular, practical knowledge as defined by Carter (1990).

After coding and analyzing the data, the mean number of representations used per lesson plan was computeted (See Figure 4). The mean number of representations used in the lesson plans increased over the course of the semester from seven to twelve. The researchers attribute this increase to the teacher candidates having acquired more comfort, fluency, and flexibility with planning to use a variety of representations as a result of their engagement in their methods course and their participation in the filed component of the course. By choosing to use more representations and moving between these representations flexibly in their lesson planning, these teacher candidates demonstrated their ability to make mathematical knowledge usable, which supports the work of Ball & Bass (2000). Thus, these learning opportunities presented in the methods course could have possibly increased the breadth of these teacher candidates' pedagogical content knowledge resulting in their articulation and inclusion of more approaches and ideas; that is, representations in their lesson plans. Certainly, learning to teach with understanding is a process that takes time, experience, and continued growth of the knowledge base of how students learn.

Closing remarks

Serving as one aspect of the framework to this study was Shulman's (1986) concept of "pedagogical content knowledge" which he defines as the ability to represent ideas in ways that are understandable to students. Considering the fact that these teacher candidates were instructed to develop lesson plans that would describe effective ways to teach a particular K-8 mathematics topic, the researchers argue that these individuals were applying their developing pedagogical content knowledge, illustrated by their choices of the various representations as defined by Lesh, Post, and Behr (1987). This supports the researchers aforementioned claim that methods courses can provide opportunities for teacher candidates to develop their pedagogical content knowledge further and, one method of observing this pedagogical content knowledge develop is through the coding and analyses of their lesson plans via the lens of representation.

Additionally, after analyzing the results of this study, the researchers do not claim that any one representation is better than another, nor do they suggest the existence of some optimal number of representations that would yield a highly effective lesson plan. The researchers offer that it is not the representation that is used, but how it is used and who uses it, whether by the teacher or the students, that seemed to contribute to the effectiveness of a lesson plan. Lesh, Post, & Behr (1987) offer that strengthening the ability to move between and among representations improves the growth of students' understanding of mathematical concepts. The researchers offer that teacher candidates need assistance in developing and strengthening this same ability, but on a metacognitive level, thereby enhancing their pedagogical content knowledge and their ability to make mathematics usable. Further analysis of representation within the context of planning for mathematics instruction coupled with teaching for

understanding will yield additional and valuable knowledge in observing and describing teacher candidates' growth in pedagogical content knowledge.

This study provides evidence of the potential benefits of incorporating the knowledge base on mathematical representations into a math methods course, as by using mathematical representations, teacher candidates can demonstrate their developing pedagogical content knowledge. After carrying out this study, Lesh, Post and Behr's (1987) research on the five mathematical representations was shared and discussed with the teacher candidates. In closing interviews, these teacher candidates shared their advocacy for including this body of research into a mathematics methods course. Some of their comments included:

> Mathematical representations could help in planning for teaching lessons in mathematics because I can have a variety of ways to show a concept.
>
> Being aware of different representations to teach would help you reach each of the students in your class in the most effective way. Not all kids learn the same.
>
> Knowing about mathematical representations would help me think outside of the box and be more creative in m lesson plans.

As stated earlier, given that a primary goal of a methods course is to prepare future teachers to teach the subject matter in effective and engaging ways such that student understanding is maximized, the researchers strongly recommend the inclusion of representation in mathematics methods courses.

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Figure 1
Researchers' interpretation of the five different representations of a mathematical idea

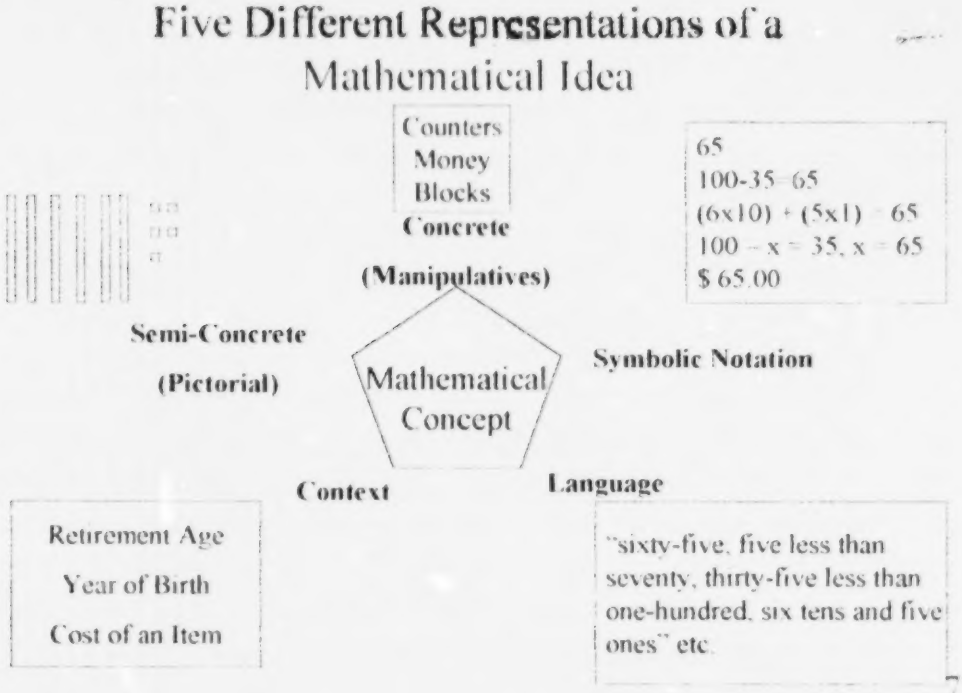

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Figure 2
Overall summary of mathematical representations by percentages

	Concrete	Language L_1 L_2	Symbolism	Semi-concrete (pictorial)	Context	Total
Initial Interval Individual LP	10	33 L_1=31 L_2=2	32	16	9	100
Middle Interval Group LP	12	31 L_1=28 L_2=3	30	18	9	100
Final Interval Individual LP	18	46 L1=39 L_2=7	14	15	7	100

Figure 3
Student vs. teacher use of representations by percentages

	Concrete (Manip.)	Lang. L_1 L_2	Symb.	Semi-concrete (pictorial)	Context	S	T	Total
Initial Interval Indiv. LP	S=7 T=3	S=7 T=26 S_{L1}=6 T_{L1}=25 S_{L2}=1 T_{L2}=1	S=13 T=19	S=5 T=11	S=2 T=7	34	66	100
Middle Interval Group LP	S=11 T=1	S=13 T=18 S_{L1}=10 T_{L1}=18 S_{L2}=3 T_{L2}=0	S=14 T=16	S=11 T=7	S=4 T=5	53	47	100
Final Interval Indiv. LP	S=13 T=5	S=19 T=27 S_{L1}=15 T_{L1}=24 S_{L2}=4 T_{L2}=3	S=8 T=6	S=7 T=8	S=3 T=4	50	50	100

Figure 4
Mean number of representations used per lesson

Initial Interval (Individual LP)	7
Middle Interval (Group LP)	12
Final Interval (Individual LP)	13

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